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Towards Industrialization of FOPID Controllers: A Survey on Milestones of Fractional-Order Control and Pathways for Future Developments

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ABSTRACT The interest in fractional-order (FO) control can be traced back to the late nineteenth century. The growing tendency towards using fractional-order proportional-integral-derivative (FOPID) control has been fueled mainly by the fact that these controllers have additional "tuning knobs" that allow coherent adjustment of the dynamics of control systems. For instance, in certain cases, the capacity for additional frequency response shaping gives rise to the generation of control laws that lead to superior performance of control loops. These fractional-order control laws may allow fulfilling intricate control performance requirements that are otherwise not in the span of conventional integer-order control systems. However, there are underpinning points that are rarely addressed in the literature: (1) What are the particular advantages (in concrete figures) of FOPID controllers versus conventional, integer-order (IO) PID controllers in light of the complexities arising in the implementation of the former? (2) For real-time implementation of FOPID controllers, approximations are used that are indeed equivalent to high-order linear controllers. What, then, is the benefit of using FOPID controllers? Finally, (3) What advantages are to be had from having a near-ideal fractional-order behavior in control practice? In the present paper, we attempt to address these issues by reviewing a large portion of relevant publications in the fast-growing FO control literature, outline the milestones and drawbacks, and present future perspectives for industrialization of fractional-order control. Furthermore, we comment on FOPID controller tuning methods from the perspective of seeking globally optimal tuning parameter sets and how this approach can benefit designers of industrial FOPID control. We also review some CACSD (computer-aided control system design) software toolboxes used for the design and implementation of FOPID controllers. Finally, we draw conclusions and formulate suggestions for future research.

INDEX TERMS Fractional Calculus; Fractional-Order PID Control; Industrial Applications; Frequency-Domain Analysis; Optimal Tuning; Fractional Control Implementation

I. INTRODUCTION

Classical PID controllers have been widely used in industrial applications and have become an industrial standard for process control. While more powerful control techniques are readily available, the popularity of the PID controller has remained due to its relative simplicity and ease of adaptability to industrial applications [2]. However, it is also commonly acknowledged that only a fraction of the existing

PI/PID controller-based loops are tuned to achieve optimal performance [3].

In recent years, the emergence of new computational techniques for fractional calculus has made possible the transition from classical models and controllers to those described mathematically by differential equations of non-integer order. Thus, fractional-order dynamic models and controllers have been introduced. In the industrial context, the apparent benefit of fractional calculus was initially justified at the process model side (see, e.g., [4]–[9]). It has proved more difficult to convey the advantages of fractional calculus on the controller side because of implementation issues.

The parallel form of the FOPID controller was introduced by Podlubny in [10] as

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + K_I s^{-\lambda} + K_D s^{\mu}, \qquad (1)$$

where $(\lambda, \mu) > 0$. Such a controller has more tuning freedom and thus a wider region of parameters that stabilize the plant under control and offer improvements in control loop robustness. Corresponding studies have been carried out to confirm this fact (see, e.g., [9], [11]–[16]), and we will glance at some milestone works of literature, addressing fractal robustness, in the following sections.

Even though FOPID controllers offer advantages over IOPID controllers, the adoption of the former in industry has been slow [17]. In this paper, we aim to explore the reasons for this slow adoption by conducting a survey of recent results related to advantages of FOPID controllers, their implementation, and industrial applications. We also focus on specific advantages of using FOPID controllers stemming from frequency-domain analysis and provide relevant conclusions. We review tuning methods for FOPID controllers that seek globally optimal solutions motivated by the plug-and-play potential of systems that are built on top of these methods. We also discuss the possible issues related to industrialization of FOPID controllers.

The rest of the paper is organized as follows. In Section II, performance and implementation comparisons of IOPID and FOPID controllers are carried out based on recent publications. Some important milestones in the development stages of FO control are summarized in Section III. Throughout Section IV, recent research related to the prospective industrial use of FOPID controllers is reviewed. In Section V, we attempt to cover the topics of FOPID controllers versus the high-order integer-order controller issue by reviewing relevant publications. In Section VI, heuristic tuning methods for FOPID controllers are reviewed. Then, in Section VII, the question of the industrialization of FOPID controllers is addressed and some related patents are discussed. Through Section VIII, useful analysis and design tools are introduced and realization issues of FO controllers are addressed. Then, future perspectives of a low-cost FO controller-embedded system realization that encourages industrial use of FO controllers are presented. Finally, in the last section, conclusions are drawn.

II. A GROWING TREND FROM CLASSICAL PID CONTROLLERS TO FOPID CONTROLLERS

Based on the feedback from some industrial partners [18], the main issue of adopting FOPID controllers in industry can be summarized in the form of the following question: "*Can the implementation complexity and cost concerns of FOPID controllers outweigh the benefits arising from the additional tuning flexibility of FO control?*" Indeed, in the most basic case, to implement a conventional PID controller digitally, one just implements the following discrete control law in software, essentially fitting into a single line of code corresponding to the equation

$$u(k) = K_p e(k) + K_i \sum_{j=0}^{k} e(j) + K_d(e(k) - e(k-1)), \qquad (2)$$

whereas for a FOPID controller, one usually must use approximations in the form of high-order conventional transfer functions that are often more complicated and thus require more computational resources, but most importantly, the FOPID controller must have been proven to be at least as reliable as the conventional PID controller. Modern embedded software solutions have been found to handle the additional implementation complexity easily [6] and are, at the same time, more energy efficient. Thus, in what follows, we focus on the benefits of FOPID controllers with respect to achievable performance improvements as well as on reliability concerns.

In general, to make an admissible comparison of IOPID and FOPID controller performance, one can consider the results of global optimization-based methods for tuning both controllers so that the best possible controller gains and orders are assumed to be obtained for performance evaluation [19]. We expand on this issue by discussing the issue of sufficiency of performance evaluation in Section 6. Here, we review some notable results.

IOPID and FOPID controllers have been designed for the control of a nonlinear boost converter using an artificial bee colony algorithm [20]. Based on simulation results, the authors conclude that the "*proposed FOPID controller can improve the startup response of the boost converter by using less on-off switching actions compared to the optimal PID controller*" and stress the practical benefit of the FOPID controller for on-off switching control. This essentially means an improvement in the control law, and even if this is the only benefit, it offers great value for industrial process control because it can help introduce a positive impact on energy efficiency and the life span of switching components by allowing for a control with less switching effort. Moreover, better rejection of disturbances and better output voltage regulation are cited as advantages, which implies improved quality of the control system and therefore improved production quality for the industry.

Further, the authors of the conference paper in [21] applied both IOPID and FOPID controllers to a liquid-level control problem while tuning both using genetic programming. The results show that *"... the PI$^{\lambda}$D$^{\mu}$ has performed slightly better for the response signal ..."*. However, one remarkable result is that a more desirable control law is also obtained in this case by measuring the variance of the control signal $u(t)$. For IOPID, this value is 3.44, while for FOPID, the value obtained is 1.39. Furthermore, the same conclusion regarding a reduction in the control effort is reached in [22]. This result can be considered very important in cases such as process systems and precision positioning systems where the generation of heat reduces precision, where reducing energy due to control effort matters. An important point here is that even slight improvement in the control law of an industrial process may have valuable impacts in control of mass production systems in terms of quality, efficiency and sustainability, especially when the manufacturing industry is viewed at scale: smaller individual benefits are then combined to form a larger overall benefit.

In [23], the design of a FOPD motion controller was investigated. One important conclusion was that "*...the inefficiency of PID controllers for simultaneously ensuring specifications [was shown] in the cases that these specifications are simultaneously achievable by an FOPD controller...*", thus showcasing the tuning flexibility of FOPID controllers compared to IOPID controllers.

Let us also consider related critical research. One interesting critical work is that of [24], which uses the term "fragility" with respect to FOPID controllers designed for FOPDT plants. In the paper, the authors assume that the parameters of the controller are subject to variation and thus devise measures to study this scenario. One practical example is when the parameters of a tuned FOPID controller are changed manually. In this case, having an idea about the fragility of the FOPID controller would be useful. However, the fragility of control performance can also be an important matter for nonideal (approximate) realizations of FOPID controllers. There is also a critical paper published in a Russian journal [25], the title of which can be translated to English as "*On the Inadvisability of using Fractional-order PID Controllers*". The main conclusion of the paper is that by applying a proper optimization procedure, it is possible to obtain IOPID controllers that are superior to the FOPID controllers obtained in [26]. However, the premise itself is based on insufficient evidence, so the conclusion, taken more generally, is questionable. Some concerns associated with statistical fairness of performance evaluations, given in case of use of heuristic optimization techniques, are also pointed out for establishment of more reliable performance comparisons in Section VI.

When assuming that the number of design specifications is lower than the number of controller parameters, a generalized robustness regions method [9] can bring a good measure of controller fragility. The method in principle results in the set of all controller parameters satisfying given requirements. Such a set is defined by the boundary in the controller parameter plane/space. Consequently, the "size" of the set is in relation to controller fragility while given robustness is ensured (e.g., gain and phase margins). Assuming the control of a unit communication channel, thus discarding contributions of plant function types to the frequency-domain response of the feedback loop, one research effort reveals contributions of the FOPID controller to the frequency response of a feedback loop, and some clues to controller performance fragilities are considered [27]. However, future research is needed at least in terms of proper initial controller selection and its exact fragility quantifications.

The advantages of FOPID controllers can be seen most clearly by invoking frequency-domain analysis and control design methods. Seminal works on the subject include [28]–[30]. The results on basic design methods were reported by Padula in [31]. One of most generic currently known methods for simple controller design was introduced in Schlegel et al. [32], allowing the computation of an admissible controller parameter region given by the upper limit of the sensitivity function. In contrast to other known research, a frequency dependent upper limit of the sensitivity function is allowed to be defined, highly beneficial for cases when the system has, e.g., oscillatory modes or when one needs to include available bandwidth in robust design (i.e., following Bode's theorem). However, future research is needed to extend the principle of the FOPI controller. In recent studies, a metric for measuring the reference-to-disturbance ratio (RDR) was proposed in [33], [34], which can be considered to evaluate disturbance rejection performance of IOPID and FOPID controllers in the frequency domain. Robust design for parametric variation of control systems can also be achieved through the assessment of maximum sensitivity properties of the controllers. A related study was conducted in [35].

To conclude this section, we now provide some comments related to FOPID controller reliability. The main concern here can be represented by the following logical question:

$$\text{Relatively complicated implementation of FOPID} \Rightarrow \text{Greater danger of erroneous performance?}$$

This question follows from the typical situation where the more complicated the system becomes, i.e., the more different components that it has, the greater the danger of malfunction of at least one of the components leading to the deterioration of the entire system [36]. From a purely theoretical point of view, it is relatively easy to confirm the reliability of computation of fractional PID controllers; see, e.g., [6]. However, to ensure that FOPID controllers are ready for at-scale deployment to industrial applications, relevant research must also be conducted to establish the necessary technology readiness level (TRL) [37] of the developed control solutions. For example, in [38], TRL $= 5$ is achieved by confirming the performance of the

tuned fractional-order controller in a series of experiments, thus confirming the reliability of the implemented control algorithm.

More research is expected to be published further confirming the reliability of FOPID controllers and reaching greater TRL levels. This additional research will likely ensure greater trust of industrial partners towards FOPID technology and thus make its industrial integration smoother.

III. SOME MILESTONES OF FO CONTROL

After two decades of academic research in the FO control domain, it became apparent that for FO control to become an industrial control standard, that is, for industrialization of FO control systems to take place, plausible cost-benefit analyses of the control performance improvements offered by FO control systems must be performed. Hence, on the way to industrialization of FO control, highlighting some practical benefits of FO control will serve as motivation for the industrial community when its representatives are looking for better and more flexible solutions for a certain type of control problems. Implementation cost of FO control systems relies mainly on the realization of basic fractional elements [39].

In this section, we focus primarily on some performance benefits of FO control in control practice that have been highlighted by field researchers in the two decades of history of related research. These benefits can be classified into three main topics:

1) Improvement of control system robustness against system parameter variations and system perturbations;
2) Contributions to optimal control of time-delay systems;
3) Contributions to disturbance rejection control.

In the following subsections, we briefly introduce relevant milestones and significant theoretical evidence regarding the topics highlighted above. Methodologically, the evidence presented in this section relies on mathematical derivations, analytical solutions, and optimization methods.

A. BEGINNING OF THE "FRACTAL ROBUSTNESS" ERA IN CONTROL RESEARCH

A remarkable advantage of FO control can be seen from the perspective of frequency-domain analysis. This advantage originates from utilization of fractional calculus, which has indeed manifested as a tool for accurate modeling by allowing orders of derivatives to be $\alpha_n \in \mathbb{R}$, which infinitely expands the concept of differential equations with regard to modeling system dynamics. Specifically, exploitation of fractional calculus increases the frequency-domain characterization options of dynamic system models by enabling fractional adjustment of amplitude and phase responses. When this fractional adjustment is applied to control, finer tuning of controllers to obtain improved control performance becomes possible. Thus, one can find concrete evidence of FO control performance improvements in frequency-domain optimal controller design tasks [40] by, e.g., employing loop shaping design techniques. To gain more insight into frequency-domain design methods of FO controllers [40], [41] and a discussion of specific advantages in terms of control system performance provided by the FO controller family, the reader is advised to study the work reported in, e.g., [27], [40], [41].

The isodamping property for control systems has been widely studied due to its potential for improving robust control performance. With an ideal realization of this property, researchers have aimed to establish a constant phase margin at gain crossover frequency because this can potentially improve the robustness and stability of control systems because sporadic gain variations of an open-loop transfer function do not lead to a phase shift in open-loop systems as a result of its flattened characteristic. Thus, a design approach that offers preservation of the phase margin under gain variations is expected to contribute to stability and robust control performance of practical control systems. This concept is referred to as "*fractal robustness*" in control literature [30], [42]–[44] and has been addressed extensively in frequency-domain optimal control design research. Useful demonstrations that address the classical isodamping property can be found in [28], [45]. A generalization of the isodamping property has also been discussed in [19].

Regarding the historical development of the concept, an ideal form of robustness against gain variations was already mentioned for negative feedback amplifier circuits by Bode in [46]. Accordingly, the ideal shape of the open-loop transfer function is typically referred to as Bode's ideal loop [30], [47]–[49] and is expressed as

$$L(s) = \left(\frac{\omega_c}{s}\right)^{\gamma}, \tag{3}$$

where ω_c is the gain crossover frequency defined at $L(j\omega_c) = 1$. The parameter γ determines the slope of the magnitude curve with a fractional slope of -20γ dB/dec on a log-log scale and the phase curve of the system with a flattened phase characteristic at the value of $-\gamma\pi/2$ rad [50], [51]. Researchers concluded that this property, associated with fractional-order dynamics, could provide significant advantages for control systems in the following way. The property presents an infinite gain margin and corresponding insensitivity to system stability to gain alterations. At the same time, phase margin is also constant for the whole frequency range (a complete flatness) and therefore is independent of gain changes. The slope of the magnitude curve and the constant phase margin can be adjusted by varying the fractional order γ.

As long as the phase margin is independent of the gain variation of a given system, the phase margin ensures preservation of system stability in case of gain variations. These properties are very prominent for robust control prospects. Due to its benefits for robust control, Bode's ideal transfer function was utilized directly in tuning problems of controllers, for instance, PID tuning according to

Bode's ideal transfer function [51]–[53]. A clear theoretical demonstration of the benefits of FO control was explained by considering Bode's ideal transfer function in [49]. This control system was composed of a fractional controller in the form of $C(s) = s^r$ and a classical "double integrator" plant function in the form of $G(s) = A/s^2$. This type of plant function can be used to model fundamental systems such as single-degree-of-freedom translational and rotational motion in robotics. A fractional-order closed-loop control system that can be represented in the form of Bode's ideal transfer function was given as

$$T(s) = \frac{A}{s^{2-r} + A}. \tag{4}$$

The main advantages of this closed-loop systems were pointed out in [49] as:

1) Gain margin is infinite. This property provides the advantages of being insensitive to gain changes by properly tuning the fractional order r.
2) The phase margin is constant at $\phi_m = \pi\left(1 - \frac{2-r}{2}\right)$: the gain variations alter the crossover frequency, but the phase margin of the system is kept at ϕ_m rad Reaching Bode's ideal loop for a generic system model could lead to more general filters that can exhibit control performance that is superior to a simple FOPI/FOPID controller. The complexity of the problem increases for cases when model uncertainty is introduced, obviously in frequency-domain design.

The robustness based on a flattened phase characteristic was considered a major asset of robust controller system design, and realization of this asset has become a central objective in numerous fractional controller frequency-domain tuning efforts. Continuous and smooth phase and magnitude adjustment options of fractional-order models have been utilized to obtain more flattening of the phase characteristic to realize the isodamping property. Oustaloup et al. proposed a fractional-order controller tuning scheme based on two important robust control characteristics [42]:

1) the iso-damping and
2) the vertical sliding form of frequency template in the Nichols chart [41].

Then, the authors demonstrated practical applications of *fractal robustness* in control [42], [43]. These efforts and their outcomes have motivated follow-up research studies where fractional-order controllers in classical control systems were investigated to measure relevant control performance enhancements [40], [54]–[57]. In these research efforts, the CRONE control scheme and its synthesis based on the desired frequency templates were studied. Control system models were suggested where the control design task revolved around the open-loop frequency response behavior of the system around [54], [58]

$$B(s) = \left(\frac{1}{\tau s}\right)^{\alpha} = \left(\frac{\omega_u}{s}\right)^{\alpha}. \tag{5}$$

A comparative introduction of CRONE and TID controllers was provided in [40], [58]. Due to the practical possibility of fractal robustness, Chen et al. demonstrated several fractional-order controller structures that can be tuned to meet the criteria of robustness for variations in the static gain of systems by using the following design constraint [45], [59]:

$$\left(\frac{\mathrm{d}(\arg(C(j\omega)P(j\omega)))}{\mathrm{d}\omega}\right)_{\omega=\omega_c} = 0. \tag{6}$$

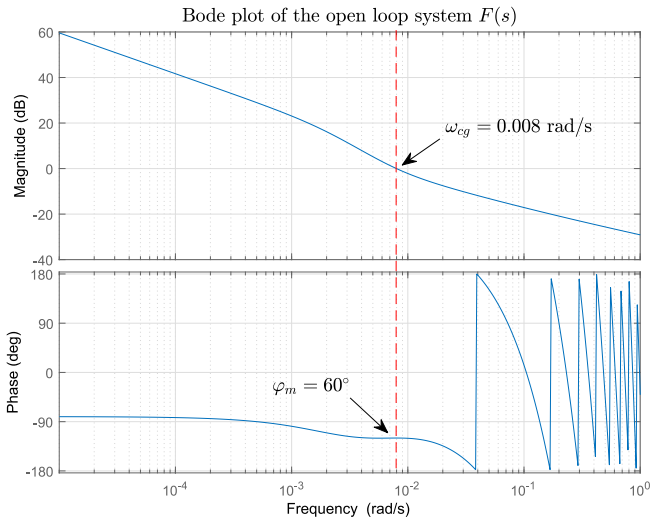
The crossover frequency ω_c is determined according to the solution of $|C(j\omega_c)P(j\omega_c)| = 1$. Clearly, the phase flatness at gain crossover frequency ω_c does not guarantee convergence to the Bode's ideal transfer function in case of complicated systems. In such a case, numerical optimization in the scope of the desired frequency band is needed.

Several frequency-domain optimal controller tuning rules were developed and compared in [35], [40]. An experimental demonstration of robust control performance improvements of FO control was presented for a heat flow experimental system and compared with classical PID control [60]. In most of the related works, impacts of the isodamping property have been demonstrated graphically by preservation of overshoots in system step responses for a gain alteration range of the system [28], [61]. Figure 1 shows results of a study that indicates a flat phase curve at the crossover frequency and corresponding robustness in the step response of the system against the controller parameter variation. In a recent study, a frequency framing method to implement a flattened phase response around the crossover frequency was introduced for optimal FOPID controller design for time-delay systems [62]. In addition, a variant of the FOPD controller, which is called a fractional-order [proportional derivative] (FO-[PD]) controller, was shown to further improve robust control performance [59]. FO-[PD] controllers were implemented in the form of fractional power of a PD controller, which allowed fractionally modifying the frequency responses of PD controllers.

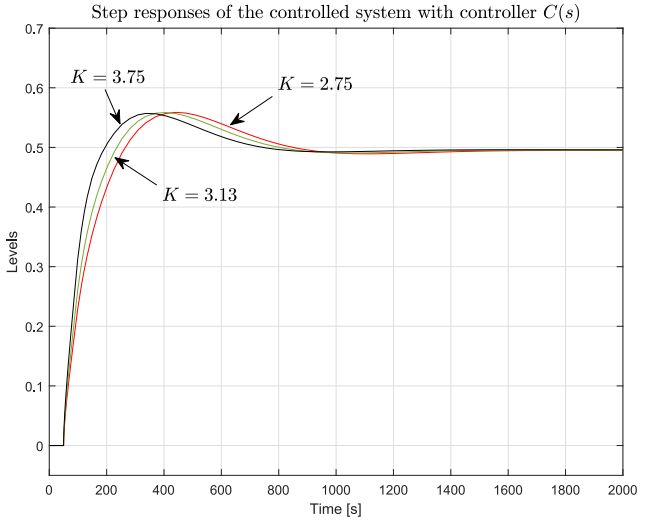
In real-world control applications, parameter variations or uncertainties commonly occur, and fractal robustness allows the minimization of the impact thereof on live industrial applications, which makes fractal robustness a very attractive property to pursue in general.

B. OPTIMALITY CONCERNS IN CONTROL OF TIME-DELAY SYSTEMS

Another benefit of FO control is manifested in the optimal control of time-delay systems. An apparent time delay is a time interval where the controlled system is assumed to give almost no response to the applied control signal, and this type of delay is sometimes referred to as system lag or system dead-time. Some real systems can exhibit very large time delays, and such large time delays can easily cause deterioration of the performance of control loops due to late arrivals of error signals to controllers. Consequently,



(a) Bode plots of a open-loop transfer function, showing a flat phase response at the crossover frequency



(b) Preservation of overshoots while changing the gain of the plant in case of an optimally tuned FOPID controller system

FIGURE 1: Illustration of the isodamping property: flatness of the phase response around the critical frequency ensures that the phase margin remains constant under changing system gain resulting in the same amplitude of the overshoot in three consecutive experiments. This example is reproduced from [28], where the controller $C(s)$ is defined by $C(s) = 0.6152 + \frac{0.01}{s^{0.8968}} + 4.3867s^{0.4773}$ and the delayed plant under control $G(s)$ is defined by $G(s) = \frac{3.13}{433.33s+1}\mathrm{e}^{-50s}$.

such large delays can severely affect the stability of closed-loop systems, which makes controller design problems much more complicated.

While Bhambhani et al. attempted to find an answer for the research question formulated as "*Will a fractional order controller help and do better [in case of time delay systems]?*" [63], a fractional M_s-constrained integral gain optimization (F-MIGO) technique was proposed to maximize the jitter margin while minimizing the integrated time absolute error (ITAE) metric to obtain optimal control for time-delay systems. When illustrating the practical utilization of the F-MIGO design technique, a further study also demonstrated the advantages of FO control for time-delay systems and the conditions that make FO control more preferable than classical PID control [64]. In the study, the contribution of a fractional-order integrator to time-delay system control was demonstrated according to a relative time-delay parameter, which was defined by

$$\tau = \frac{L}{L+T}, \tag{7}$$

where the time delay L and the time constant T are determined based on a classical first-order time-delay plant model given in the form of

$$G(s) = \frac{K}{Ts+1}\mathrm{e}^{-Ls}, \tag{8}$$

where the term e^{-Ls} signifies the input-output delay of the system. This delay model represents an input-output lag of L seconds when the system is actuated. Larger time delays complicate controller design tasks and cause a decrease in control performance. The resulting analysis is very useful to depict impacts of a fractional-order integrator in optimal tuning problems of time-delay systems. The relative time constant takes a value in the range of $[0, 1]$. By considering different values of τ, control systems were classified as follows:

1) a delay-dominated system when $L \gg T$ and
2) a lag-dominated system when $T \gg L$.

For optimal tuning of the FOPI controllers in the form of

$$C(s) = K_p + \frac{K_i}{s^\alpha}$$

optimal values of the fractional order α with respect to the delay τ were suggested according to results of optimal tuning as

$$\alpha = \begin{cases} 1.1, & \tau \geqslant 0.6 \\ 1.0, & 0.4 \leqslant \tau < 0.6 \\ 0.9, & 0.1 \leqslant \tau < 0.4 \\ 0.7 & \tau < 0.1. \end{cases} \tag{9}$$

Noninteger values of α in Equation (9) evidently confirm the favorable application of fractional-order control to the problem of establishing optimal control for time-delay systems. These useful theoretical findings clearly indicate the practical use of FO control and contribute to ongoing discussions under which conditions of FO control can be more useful than classical integer-order control.

The findings in equation (9) verify that optimal control of time-delay systems requires fractional-order control when the parameter τ is out of the range $[0.4, 0.6)$. An optimal order α of the fractional-order case is needed for larger

ranges of τ ($0.1 \leqslant \tau < 0.4$ and $\tau \geqslant 0.6$). This point is strong evidence for control performance improvements provided by FO control for time-delay systems. Moreover, the single optimal value $\alpha = 1.1$ for a larger range $\tau \geqslant 0.6$ can be interpreted as a clue for control performance robustness against deviations. In another work, contribution of FO control to robust performance in the case of plant model uncertainties and time-delay variations was addressed, and useful results were discussed [65]. Some other studies deal with systems having variable delays. The variable delays inherently arise in networked control systems because of fluctuations in the bandwidth of communication systems [66], [67].

Consequently, solvability of optimal tuning problems under strict robustness constraints can be viewed as theoretical evidence for contributions of fractional-order controllers to optimal control problems. At this point, it can be useful to mention another interesting study wherein the achievability of frequency-domain requirements (phase margin, flattened phase (isodamping), and gain crossover frequency for robust control) was discussed [23].

To facilitate practical industrial use of FO controllers in process control, there is also demand for future research efforts that simplify FO controller parameter tuning by providing suitable gain/order tables or even analytical functions. These design simplifications for industrial use of the FO controller can be developed so that its settings are parameterized by at least two tuning knobs: maximal value of sensitivity function and available loop bandwidth.

As an example of an effort directed at producing "rules of thumb" for FOPID controller tuning in industrial applications, one can consider the work in [68], [69]. Further similar research should be conducted to match that available for the conventional PID controller [3].

C. DISTURBANCE REJECTION PERFORMANCE OF FO CONTROL

Disturbance rejection is one of the most important requirements of practical control systems so that if the real-life control system is exposed to environmental disturbances that are generally of an unpredictable nature, the control system will still perform as specified by the designer. A practical robust control system should be able to deal with negative impacts of environmental disturbances on the control performance. Therefore, to reach a desired disturbance rejection performance in case of an additive output disturbance model, frequency-domain tuning schemes of FO controllers frequently employ a sensitivity function constraint expressed as [28]

$$\left| \frac{1}{1 + C(j\omega)P(j\omega)} \right| \leqslant B, \forall \omega \leqslant \omega_s. \tag{10}$$

Disturbance rejection performance improvements of different FO controllers employing the sensitivity function constraint were discussed in [64]. In another study, robustness against gain variations and load disturbance was demonstrated for FOPID control of parabolic distributed solar collectors, and results were compared with classical PID control in [70].

For consideration of additive input disturbance rejection models, the RDR index was proposed in [1], [33], [34] as

$$\mathrm{RDR}(\omega) = |C(j\omega)|^2 \tag{11}$$

to measure the disturbance rejection capacity of a closed-loop system. Improvement of input disturbance rejection performance of fractional-order control systems is made possible by increasing $\mathrm{RDR}(\omega)$ values in the low-frequency range. A recent study has numerically investigated control dynamics of a FOPID controller relative to PID controller dynamics, and contributions of the FOPID controller to disturbance rejection capacity were discussed from a plant-independent analysis point of view [27].

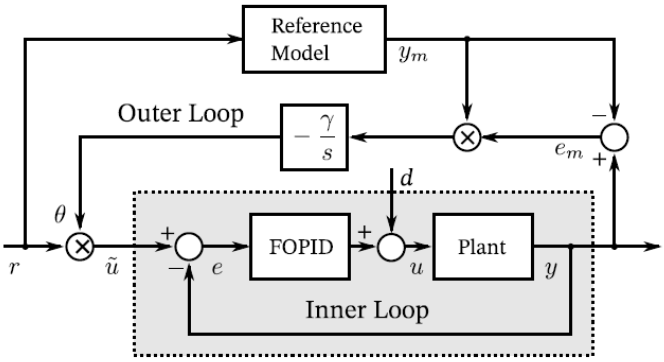
To further improve the performance of existing FOPID control loops, a multiloop model reference adaptive control (MRAC), which is a hierarchical integration of the FOPID control loop and the model reference adaptive control loop, was proposed, and control performance improvements of such multiloop MRAC- FOPID structures were shown for fault-tolerance control in [71] and disturbance rejection control of a magnetic levitation system in [72] (see Figure 2). This study also demonstrates that this multiloop structure improves the control law of FOPID control systems. A comprehensive survey of FOPID controller tuning methods and advanced control methods with FO controllers can be found in [73].

Another emerging advanced topic of fractional-order systems is in the networked control field, which brings a control problem of considering large-scale systems involving distributed and decentralized control elements in a network, for example, a collection of sensors and actuators that are connected via a communication network. Boukal et al. addressed the stability concern and decentralized observer design of large-scale fractional-order systems. The outcomes of the work have the potential of contributing to networked control of fractional-order systems [74]. Application examples of the networked control for cyber-physical systems are discussed in [75], [76].

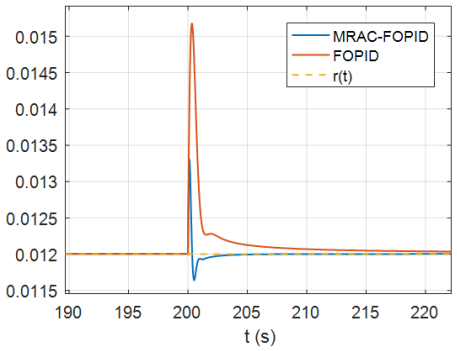
IV. INDUSTRIAL APPLICATIONS OF FOPID CONTROL

The main area of application of FOPID controllers is in process control (e.g., liquid-level control) and power systems (e.g., boost converter) due to the principal interpretation of the FOPID controller: in essence, the FOPID controller is a band stop filter when one can define its shape and steepness in a more flexible way than using an IOPID controller.

In mechatronics, the application of FOPID seems to be limited to a rigid structure that could, however, result in beneficial applications in, e.g., automotive driving (car platooning), where vehicles can be considered simple point-masses that are virtually interconnected. Moving into the area of robots and machines, the complexity of controllers grows to higher orders because the increased elasticity



(a) Block diagram of multiloop MRAC and FOPID structure



(b) A comparison of the disturbance rejection performance of a multiloop FOPID structure and conventional FOPID control from simulation of a magnetic levitation system [72].

FIGURE 2: Multiloop control using a FOPID controller wherein the main control loop receives an additional loop that ensures robust performance under the effect of unknown disturbances

of mechanical structures brings additional resonances and antiresonances that currently overlap with the desired control loop bandwidth. This trend is caused by machines being designed to be lighter and work faster, i.e., with increased bandwidths. Although fractional PID controllers are also typically implemented as high-order filters, the high-order controllers should be designed and optimized as general filters in state space considering information from all available sensors. Typically, the high-order controllers can handle up to 3 resonance and antiresonance modes of the mechanical structure within the relevant frequency band. If one wants to preserve controller simplicity, multi-input, multi- output (MIMO) structures should be used [77]. Such problems, clearly, are tricky to handle by single-input, Single-output (SISO) FOPIDs. Hence, in mechatronics, the FOPID applicability seems to be limited to rigid structures, equivalent simplified models, or low-level drive control.

Let us investigate some promising applications in more detail.

With respect to industrial use of FOPID controllers, Smith predictor-based fractional-order-filter PID controllers designed for long-term delay systems was proposed in [78], and a survey paper addressing applications of FO control to well-known control elements is found in [79]. The main content of the paper comprises the description of various classical control techniques (PID control, sliding-mode control, backstepping, MRAC) extended to make use of fractional-order calculus. There is a single example provided of a control problem where a control system is developed for a cement mill.

In [80], a fractional controller is designed for a *wind turbine generator*. Here, based on simulations, the authors claim the following: (1) *"... while the fractional order PI controller ... properly tracks the input command, the simple integer order PI controller is not capable to cope with the nonlinearity due to backlash phenomenon.*" (2)*"... The fractional order control system accurately tracks the reference input [under plant parameter variation]. However, the integer order control system becomes unstable [under the same conditions*]", once again highlighting the advantages of FOPID controllers.

In [81], a FOPI controller is developed for an *electro-hydraulic system*, with a particular emphasis on the energy saving aspect. To validate the performance of the control system obtained, a PI controller was designed, and experiments with both PI and FOPI controllers were conducted (including variants thereof where an *orifice compensation* (OC) system was added). The authors claimed that the "*the FOPI+OC method has the minimum tracking error, and common PI controller has the biggest one.*"

The authors of [15] design FOPID controllers for *twin rotor systems*. Here, the authors evidently show that *"... disturbance rejection by applying the [set-point weighted FOPID] controller is always better done in comparison with the designed [setpoint weighted PID], [filtered FOPI] and [filtered PI] controllers ...".*

In [82] the impact of FO methods on *tilt control of rail vehicles* is discussed. Here, both the design of FOPID controllers and the reduction in corresponding approximations are investigated. The results of FOPID control are compared to that achieved with conventional PID controllers. To cite the authors, *"[Related figure] illustrates the immense benefit of fractional order based control on improving tilt following (with full order control)."*

In [83], FOPI controllers were designed for *industrial electrical drives*. The paper deals with a particular design method. To cite the authors directly, "*It is remarkable that the fractional pre-filters almost cancel the oscillations. The*

improvement is even more relevant in the case of speed control. The overshoot is greatly reduced and the settling and rise times are also reduced with respect to the PI-controlled system."

In [84] the use of fractional control of unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs) was investigated. FO control has been applied to the problems of trajectory tracking, attitude control, path planning, state estimation, formation control, fault-tolerant control, collision avoidance, fault detection and diagnosis and has been found to be usable in all these areas, though applications are obviously limited where specifically FOPID-type controllers are concerned. However, implementation of FO controllers is once again cited as a serious issue.

In [85], a *precision positioning system* basis of FO control was designed and analyzed. Two cascaded control loops with decoupled SISO controllers were implemented for a moving mass controlled on a mass-spring system that could be modeled as a fourth-order system. Applying a fractional-order derivative compared to an integer-order derivative, the bandwidths were extended by 14.6% and 62%, for the inner and outer loops, respectively. A closed-loop positioning bandwidth of the wafer at 60 Hz was achieved, resulting in a positioning error of 104 nm, which was limited by sensor noise and pressure disturbances.

Furthermore, fractional-order calculus has been utilized for enhancement of extremum-seeking control systems for several applications. Some recent studies on fractional-order extremum-seeking control (FO-ESC) have described applications of FO-ESC in lighting system control [86], [87], a dynamometer system [88], and maximum power point tracking [89].

There is clear evidence that fractional control has benefits for industrial applications based on the reviewed material. More research in the area of industrial application of FOPID controllers must be conducted to ensure smooth transition from theoretical contribution into practice.

V. FOPID CONTROLLERS VERSUS HIGHER-ORDER INTEGER-ORDER CONTROLLERS

For real-life implementation of FOPID controllers, higher-order IO transfer function approximations are generally used [29], bringing about the question: *"If integer-order approximations are used anyway, why not just use high-order integer-order controllers instead of FOPID approximations?"* Although this is a very important issue from the modeling standpoint, surprisingly few publications deal with this matter. A few (almost) relevant papers are reviewed below.

In [90], the author proposes some rules for selecting the parameters for approximating FOPID controllers using the Oustaloup recursive filter method, potentially helping to reduce the order of the resulting controller. However, this approach does not solve the *"FOPID versus high-order IO controller"* issue that is the topic of this section. In one instance, the authors of [91] suggest using FO models for *"compressing"* high-order integer-order models. The proposed method has certain limitations (e.g., it works only for real transfer function poles). This approach is extended in [92]. Here, the authors use fractional-order models for creating compact models obtained by (1) identifying a high-order integer-order model; (2) converting the model to a fractional-order model; and (3) optimizing the fractional-order model. All of these contributions propose the basic idea that FO models can be used for compact description of high-order systems, but this fact will not end ongoing discussions on the importance of FO control for control practice. Clearly, more relevant research efforts addressing this issue are required. As the current conclusion, we propose the following argumentation:

1) FOPID controllers are *not* generally equivalent to high-order integer-order controllers, and we also argue that a near-ideal realization of a FOPID controller can be made available by applying mixed-mode digital analog system design. This design can use near-ideal realization of analog fractional-order elements (FOEs) that can be integrated by a digital circuit to implement more complex fractional-order system structures. Some details on realization of fractional-order systems are provided in Section VIII.
2) Currently, the control community focuses on a FOPID controller being an extension to an IOPID controller, and they treat its two additional parameters as "*tuning knobs*". However, the implementation of a FOPID controller relies on the use of a high-order IO approximation and, hence, potentially gives rise to even more tuning knobs in the form of all of the parameters of the resulting approximated system.

Nevertheless, there is a difficulty in managing (e.g., tuning) all the parameters of a high-order integer-order controller, which is more difficult than in case of a FOPID controller.

Perhaps another issue here is the necessity to provide a coherent interpretation of the different forms of the FOPID controller used for actual implementation. That is, we have the following two cases as seen from the continuous time modeling perspective:

1) Parallel form of the PID controller is represented and implemented by

$$C_{PID}(s) = K_p + K_i s^{-1} + K_d s \Leftrightarrow \quad (12)$$
$$\tilde{C}_{PID}(s) \approx K_p + K_i s^{-1} + K_d \tilde{s}_d$$

since the differential component can only be implemented as an approximation $\tilde{s}_d$. The implementation is exact if the differential component is omitted; in control engineering practice, this is usually the case, as most industrial controllers are of the PI type [18]. Typically, (12) is implemented digitally, so the discrete time form of the transfer must be used, and the corresponding transform is trivial.

2) The parallel form of the FOPID controller is represented and implemented by

$$C_{FOPID}(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} \Leftrightarrow \quad (13)$$
$$\tilde{C}_{FOPID}(s) \approx K_p + K_i \tilde{s}_i^{-\lambda} + K_d \tilde{s}_d^{\mu},$$

where we now have two approximations of order n_i and n_d, typically $n_i = n_d = N$ and $N \geqslant 11$, of the form

$$\tilde{s}_i^{-\lambda} \approx \frac{b_{m_i} s^{m_i} + b_{m_i-1} s^{m_i-1} + \cdots + b_0}{a_{n_i} s^{n_i} + a_{n_i-1} s^{n_i-1} + \cdots + a_0} \quad (14)$$

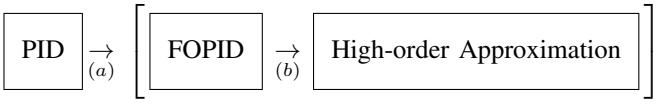
and

$$\tilde{s}_d^{\mu} \approx \frac{b_{m_d} s^{m_d} + b_{m_d-1} s^{m_d-1} + \cdots + b_0}{a_{n_d} s^{n_d} + a_{n_d-1} s^{n_d-1} + \cdots + a_0} \quad (15)$$

of the integrator and differentiator, respectively.

From the point of view of an engineer, complications arising from the implementation of (13) are not justified compared to (12), not only because of use of high-order IO controllers but also because this configuration is not intuitively understandable and cannot be explained well to actual control engineers working with specific industrial problems.

Therefore, we provide a proposal that can be viewed as a kind of consensus between using conventional PID controllers, fractional PID controllers, and high-order approximations of the latter. The idea is to establish a natural flow from the idea of simplicity of conventional PID all the way to the complexity of high-order approximations while providing a coherent justification for the increase in complexity versus the advantages that this configuration provides. The flow, then, can be represented by the following diagram:



The transitions and the corresponding justifications are as follows:

(a) The conventional PID is replaced with a FOPID controller adding two additional parameters that can be used to tune the performance of the control system to achieve the desired specifications. This transition is natural and justified since we are only improving an already known and established solution that is the PID controller. In some cases, changing the default orders $\lambda = \mu = 1$ may not even be required. Additionally, an interesting method for this transition exists, specifically, the PID/FOPID retuning method described in [39], [72]. With this method, instead of replacing the PID control system, an additional loop is introduced that converts the PID control system into a FOPID control system.

(b) We know that the FOPID controller implementations are based almost exclusively on approximations, so this transition is necessary in any case. However, the reason for including it explicitly in the diagram above is as follows. Not only can we use high-order approximations for implementing FOPID controllers but we can also work with the obtained approximations independent of the original FOPID controller implementation. In other words, the approximation can become a controller in its own right. Then, we can collect its parameters (essentially, zero and pole locations) and additionally tune them to achieve the desired control system performance.

While item (b) is clearly justified, it is also obvious that tuning many parameters is not a manual task, nor is it a plug-and-play solution. Thus, an effective method must be applied for automatic tuning of the high-order controller. In the following section, we provide a solution to this problem in the form of heuristic optimization methods that can not only tune FOPID controller parameters but are also suitable for handling large-scale tasks such as that described above.

VI. HEURISTIC OPTIMIZATION METHODS IN THE TUNING OF FOPID CONTROLLERS

Heuristic optimization techniques are commonly employed to solve high-complexity optimization problems. The versatility of heuristic search originates from its property of *perceived optimal* branching that is used to find an optimal solution to a given problem. This property makes heuristic optimization techniques a straightforward tool even for online tuning of parameters of real systems. Since industry is interested primarily in shortcut techniques to solve the practical problems, heuristic optimization techniques can provide relevant low-cost and effective solutions. To deal with computation complexity arising due to fractional calculus, metaheuristic methods are preferable to find an acceptable solution for optimal parameter values of FO systems [93].

The growing number of publications related to heuristic optimization is a tell-tale sign of the relevance of the subject. Specifically, we are interested in FOPID controller tuning. A summary of relevant papers is provided in Table 1, which presents a taxonomy of some milestone works chosen from a large number of published works. Such pervasive interest in control literature is a clear indication of benefits of employment of heuristic optimization techniques in FO controller tuning and beyond.

Initially, analytical solutions to optimal FOPID controller design problems were restricted and were based largely on the first-order model description of the plant to be controlled. This limitation was caused by mathematical complications involving analytical solutions of fractional-order differential equations. Then, Cao and Cao demonstrated the ease of adoption of PSO and GA algorithms for solution of optimal FOPID control problems to deal with those complications, and they concluded that "*...the parameters optimization of fractional order PID controller based on modified PSO is highly effective*" [93]. This conclusion has motivated numerous works that adopted di-

verse metaheuristic methods as solutions of optimal FOPID controller design problems for several control applications. Since then, design complications of FOPID controllers have been relieved from the point of view of applications. This process has paved the way for application-based analysis and comparisons of performance improvements that were provided by FO controllers: Zamani et al. reported automatic voltage regulator application that *"... concluded from the above simulations that the proposed FOPID controller has more robust stability and performance characteristics than the PID controller..."*. In another application-centered study that considered control of photovoltaic inverters for solar energy harvesting, Yang et al. reported that "... *FoPID control can obtain a lower fitness function than that of PID control owing to the use of fractional-order mechanism. Last, Po-FoPID control has the lowest fitness function; thus, it has the best control performance among all controllers.*" [94]. The majority of research studies, which provide a source of comparison data with other controllers, have indicated improvements of control performance when FO control was used in their applications. However, due to the pseudo-random nature of metaheuristic search methods, to make research findings more solid and consistent, a standard statistical evaluation process for controller performance is required. This topic will be considered in further sections.

Generally, during the optimization process, a single objective function can be preferred to achieve specific control objectives such as error minimization, rising time, settling time, gain margin, phase margin and other system specifications. However, spectacular contributions of metaheuristic optimization methods to FO control applications become more apparent and rather indispensable in multiobjective design efforts because multiobjective design is the only coherent way for obtaining more realistic and application-oriented controller designs that can better fit requirements and constraints of control applications. The improved control performance of multiobjective design optimization has been evidenced with many studies in the literature.

Meng et al. presented a multiobjective FOPID controller design by using GA to fulfill certain specifications simultaneously, including robust performance, frequency margin, overshoot and rise time [95]. Tehrani et al. demonstrated an adoption of a multiobjective evolutionary algorithm, the so-called strength Pareto evolutionary algorithm, for FOPID controller design of a boost converter, and they stated that the "... *FOPID controller has a good dynamic response along with an excellent start-up response...*" [96]. These results validate the contribution of multiobjective FOPID control design to meet application-specific design requirements. In [97], Pan et al. performed extensive performance tests on FOPID controllers and PID controllers. Several controllers were tuned for controlling an automatic voltage regulator application by using the nondominated sorting genetic algorithm II (NSGA II) with several contradictory objective functions. In this noteworthy work, the authors reported that "*For the contradictory objectives of set point tracking and load disturbance rejection, the FOPID controller is better than the PID. On the other hand for set point tracking and lower control signal, the PID outperforms the FOPID.*" and indicated that the perceived advantages of the controller depend on the objectives, so a full benefit for all objectives may not be possible for a single controller type. Frequency-domain FOPID design specifications, which were used in loop-shaping design, were also solved by using a differential evolution algorithm [98]. Plant model limitation of analytical loop-shaping methods can be exceeded by means of metaheuristic methods. In [99], an evolutionary multiobjective optimization algorithm, the NSGA II, improved with a chaotic Henon map, was used to optimize a FOPID controller in the frequency domain with contradictory objectives. Gad et al. utilized a multiobjective genetic algorithm for the fractional-order PID controller of semiactive magnetorheologically damped seat suspension, and frequency-domain design objectives were achieved; the authors stated, based on simulation findings, that "... *the proposed FOPID controller of MR seat suspension offers a superior performance of the ride comfort over the integer controllers.*" [100]. Another study presented a FOPID controller design method based on an improved multiobjective external optimization algorithm for an automatic voltage regulator system, and performance comparisons with a PID controller were discussed by considering several metaheuristic optimization methods [101]. In [102], the authors optimized FOPID controller parameters for improving power system stability according to performance indices that include integral absolute error, integral squared error, integral of the time-weighted absolute error and integral of time multiplied by the squared error.

Some recent studies, which can be useful for multiobjective FOPID controller design for improvement of external disturbance rejection, can be summarized as follows: A FOPID tuning method for disturbance rejection control by using an RDR index in a multiobjective big bang big crunch optimization algorithm was presented in [103]. In another study, a variant of the Pareto optimal design strategy, which performs consensus boundary formulation in basic random searches, was used to achieve higher RDR performance while preserving the lowest mean square error. Performance of disturbance rejection FOPID controller design was numerically shown for a nonlinear TRMS model [104]. An interesting application of the disturbance rejection property was used by Zamani et al. to adjust the contact force of piezoelectric friction dampers for semiactive control of base-isolated structures during far-field and near-field earthquake excitations, where a multiobjective cuckoo search optimization approach was employed for FOPID tuning [105].

A. A DISCUSSION ON THE ADVANTAGES AND DISADVANTAGES OF HEURISTIC OPTIMIZATION METHODS IN CONTROLLER TUNING

An abundance of heuristic tuning studies in the FOPID controller literature indicates the applicability potential of

FOPID controllers for industrial control applications. The strength of this class of algorithms comes from their straightforward yet effective search mechanisms to find enough good solutions even for highly complex multi-objective and constrained optimization problems that can be encountered in the design of realistic control systems. Hence, it is relatively easy to adopt this sort of search algorithm to deal with mathematical complications behind optimal FO control design problems.

As a near-future perspective for industrial use, it will be possible to use heuristic optimization methods in the realization of plug-and-play-type FOPID controller devices that can be implemented on inexpensive programmable control cards. In Section VIII, we address some concerns associated with the need for automated design and realization environments to facilitate the industrialization of FO controllers.

Going beyond FOPID controllers, heuristic search can also be very useful in tuning high-order integer-order controllers. Specifically, we start with a FOPID controller approximation and allow fluctuation in the locations of the poles and zeros of the approximation, which can potentially improve control system performance beyond the capacity of a FOPID controller.

However, a lack of statistical evaluations for the results of metaheuristic methods leads to the situation where a substantiated comparison of controller performance is not possible; that is, single-run metaheuristic tuning does not provide concrete evidence that establishes the superiority of FOPID controls over conventional PID controllers. As a result of the random search nature of heuristic optimization methods, the objectivity of assessments based on single-run results is somewhat questionable: one cannot openly claim that nonstatistical single-run analysis ensures either repeatability of results or produces global optimal solutions. These methods may yield new results at each run of the algorithms, and the single-run approach may find a local solution when it is stuck at a local minimum.

Considering the nature of heuristic tuning, a statistical consideration of optimization results is apparently necessary. Otherwise, the findings of these studies are not solid and consistent. Therefore, when using results of metaheuristic optimization algorithms for controller comparison purposes, it is evident that consideration of an average value of performance indices, revelation of the standard deviation of the results, and presentation of the best and the worst designs according to statistically meaningful repeated tests are very important for fair comparisons in technical publications. Table 2 lists some statistical parameters that can be recommended for consideration in comparative works when metaheuristic optimization methods are used. Such basic statistical analysis will considerably increase the objectivity and consistency of the findings of research studies that involve heuristic optimization algorithms.

However, as we have discussed previously, yet another strategy of ensuring superior FOPID control performance can be to properly choose the control system specifications, i.e., cost functions. For example, in studying Section IV, one can observe that superior performance with respect to improvement in the quality of the control law arising from use of FOPID controllers is reported rather frequently. Therefore, one can construct corresponding cost functions taking these specifics into account.

VII. INDUSTRIALIZATION OF FOPID CONTROLLERS

The industrialization of controllers is currently quite a challenge in a competitive world where companies tend to invest resources to bring down the cost of goods and improve the benefits of the products to the customers. As has been demonstrated, FOPID controllers have technical advantages over their integer-order counterparts, but the cost of producing such controllers and the cost-benefit that the end user would realize is still something that should be investigated. Where the FOPID controller shows better technical performance in a twin rotor system in [15], the better technical performance does not necessarily mean that the cost of creating such FOPID controllers for commercial or industrial use is similar to the cost of ordinary PID controllers. In such a case, it is suggested to examine how to industrialize FOPID controllers considering financial factors. This question is open and needs to be addressed in future research.

Patents are seen as important factors in the industrialization process. Therefore, we investigate patents related to fractional-order control and implementation thereof.

The TID controller patent in [106] describes a *tilt-integral-derivative* controller similar to a PID controller, but the proportional gain is replaced with the *tilt* component that has a transfer function of $s^{-1/n}$. The advantage of this controller was explained in [55] as *"The resulting transfer function of the entire compensator more closely approximates an optimal loop transfer function, thereby achieving improved feedback control performance. Further, compared to conventional PID compensators, the TID compensator allows for simpler tuning, better disturbance rejection, and smaller effects of plant parameter variations on the closed-loop response"*.

Apart from the patents listed above, some US-based patents can be found. In [107], an invention implementing noninteger (i.e., fractional) systems is described and is based on artificial neural networks. In patent [108], a tuning method for fractional controllers is proposed. Finally, in [109], a fractional-order capacitor is described. This latter patent could be very important in the sense of achieving an ideal implementation of FO operators, and, as a direct consequence, of FOPID-type controllers.

Several worldwide patents have been registered that list several members of the CRONE team—the originators of the Oustaloup method and several generations of CRONE controllers—as inventors. Several of these patents are reviewed next.

TABLE 1: Classification of some important published works on the topic of metaheuristic optimization of FO controller coefficients

FO Controller	Meta-heuristic Method	Control Application	Comparison with PID	Statistical Evaluation	Comparisons with other Optimization Methods	Ref
FOPID	Particle Swarm Optimization	FOPID controller design for a Mathematical Model	No	No	GA	[93]
FOPID	Particle Swarm Optimization	FOPID controller design for an Automatic Voltage Regulator	Yes	No	No	[111]
FOPID	Improved Electromagnetism-like Algorithm	FOPID controller design for a Second order System with a Time Delay	No	No	GA	[112]
FOPID	Self-Organizing Migrating Algorithm (SOMA)	FOPID controller design for an Integer-Order and Fractional-Order Plant	Yes	No	No	[113]
FOPID	Particle Swarm Optimization	FOPID controller design for an Integer-Order and Fractional-Order Plant	Yes	No	No	[114]
FOPID	Modified Invasive Weed Optimization Algorithm	FOPID controller design for a Real-life Analog Plant	No	Yes	PSO and GA	[115]
FOPID	Evolutionary Algorithm	FOPID controller design for Full Vehicle Nonlinear Active Suspension Systems	No	No	No	[116]
FOPID	Genetic Algorithm	First-order system with time delay	Yes	No	No	[117]
FOPID	Modified Artificial Bee Colony Algorithm	FOPID controller design for a Process Plant Transfer Function	No	No	PSO and GA	[118]
FOPID	Chaotic Ant Swarm	FOPID controller design for an Automatic Voltage Regulator	No	Yes	PSO and GA	[119]
FOPID	Artificial Bee Colony (ABC)	FOPID controller design for Speed Control of a Chopper Fed DC Motor Drive	Yes	Yes	No	[120]
Fuzzy FOPID	Fruit Fly Optimization Algorithm	Fuzzy FOPID controller design for an electronic throttle	Yes	No	No	[121]
FOPID	Particle Swarm Optimization (PSO)	FOPID controller design for an Automatic Voltage Regulator	Yes	No	Chaotic Ant Swarm	[111]

TABLE 1: Classification of some important published works on the topic of metaheuristic optimization of FO controller coefficients

FO Controller	Meta-heuristic Method	Control Application	Comparison with PID	Statistical Evaluation	Comparisons with other Optimization Methods	Ref
FOPID	Imperialist Competitive Algorithm	FOPID controller design for load frequency control	Yes	No	No	[122]
FOPID	Improved Artificial Bee Colony Algorithm	FOPID controller design for an Automatic Voltage Regulator	Yes	Yes	PSO and GA	[123]
FOPID	Memetic Algorithm	FOPID controller design for an Automatic Voltage Regulator trajectory control	Yes	Yes	GA, PSO, ABC, DE, SOMA, PSO	[124]
FOPID	Adaptive Particle Swarm Optimization Algorithm	FOPID controller design for a typical nonlinear system	No	No	DE and PSO	[125]
FOPID	Gases Brownian Motion Optimization (GBMO)	FOPID controller design for a Two-area power system model	Yes	No	No	[126]
Fast Fuzzy FOPID	Improved Gravitational Search Algorithm	Fast Fuzzy FOPID controller design for a pumped storage hydro unit	Yes	No	No	[127]
FOPID	Gravitational Search Algorithm based on the Cauchy and Gaussian Mutation	FOPID controller design for a pumped storage unit	No	Yes	PSO and GA	[128]
fractional-order fuzzy precompensated FOPID	Hybridized ABC-GA	Fuzzy FOPID controller design for a 2DOF robot manipulator	No	No	No	[129]
FOPID	Stochastic Inertia Weight Particle Swarm optimization algorithm and Radial Basis Function Neural Network	FOPID controller design for a time-delay system	Yes	No	PSO	[130]
Fuzzy FOPID	Particle Swarm Optimization algorithm with Improved Dynamic Parameter Selection	Fuzzy FOPID controller design for a combined cycle power plant using	No	Yes	GA, ABC, PSO, DE	[131]
FOPID	Modified Artificial Physics Optimization	FOPID and PID controller design for an integer- and fractional-order plant	Yes	Yes	DE and SOMA	[132]

TABLE 1: Classification of some important published works on the topic of metaheuristic optimization of FO controller coefficients

FO Controller	Meta-heuristic Method	Control Application	Comparison with PID	Statistical Evaluation	Comparisons with other Optimization Methods	Ref
FOPID	Salp Swarm Algorithm	FOPID controller design for automatic generation control	No	No	Whale Optimization Algorithm, PSO, Teaching Learning Based Optimization	[133]
FOPID	Continuous State Transition Algorithm	FOPID controller design for a Fractional-order process plant function	Yes	No	DE and PSO	[134]
Perturbation observer-based FOPID	Yin-Yang-Pair optimization	FOPID controller design for photovoltaics inverters for solar energy harvesting	Yes	Yes	Sliding Mode Control and Feedback Linearization Control	[94]
FOPID	Moth-flame optimization algorithm	FOPID controller design for plants with model reference adaptive control	No	No	Tabu Search Method	[135]
FOPID	Modified Gray Wolf Optimization Technique	FOPID controller design for three Area nonlinear power system	Yes	No	No	[136]
FOPID	Colliding Bodies Optimization	FOPID controller design for some plants	No	Yes	PSO, DE, GA	[137]
FOPID	Firefly Algorithm with Dynamic Control Mechanism	FOPID controller design for a chopper fed DC motor drive	No	Yes	ABC, PSO, DE, GA	[138]
FOPID	SMDO	FOPID controller for TRMS	Yes	No	No	[139]
FOPID	Nelder-Mead Algorithm	FOPID controller design for Integer- and fractional-order plus time delay models	No	No	No	[140]
FOPID	Tabu Search-Based Algorithm	FOPID controller design for some plants	Yes	No	DE	[141]
FOPID	Master–Slave Stochastic Optimization	FOPID controller design for TRMS system	No	Yes	No	[142]
FO[PI]	Modified Artificial Physic Optimization	FOPID controller design for multiple time-delay systems	No	No	SMDO	[143]
FOPID	PSO, ABC	Unstable and integrating systems with time delay	Yes	No	PSO, ABC	[144]
FOPID	PSO, GA	Robot trajectory control	No	No	PSO, GA	[145]

In [110], a method for controlling the angular speed of an electromechanical drive train with low damping is described. Here, speed control of a rotating load driven by the electromechanical drive train comprises an observer to approximate the angular speed, removing in this way the need for measuring the angular speed and a noninteger CRONE-type controller to tackle various nonlinearities, thereby making the control system robust. In prior art, the authors show the drawbacks of classical PI/PID controllers particularly relevant to this application and later explain the advantages of CRONE control. This patent describes an actual industrial application of fractional-order control, which is kept active in several patent jurisdictions and hence must be successful.

However, in [146], a patent that involves a more general and software-related invention is described. At the heart of the invention once again lies the CRONE controller. While the patent had the potential to introduce fractional-order control to industrial applications in the general sense, it has apparently not been so successful since it has been kept active in the WO (world) and DE (German) jurisdictions but has expired elsewhere.

In areas that concern fractional control, several Chinese patents can also be found and are reviewed in what follows. As these patents are fairly recent as of 2019, their impact on actual industrial applications remains to be seen.

In [147], the authors develop an active magnetic bearing control method based on a fractional-order model. Closed-loop identification is employed to establish the fractional-order model of the plant. The authors then design a fractional-order PID controller and implement control of the magnetic suspension bearing system using the Oustaloup method. Concerning this particular application, the approach described in this patent correlates with findings reported in [72].

In [148], a method for the control of the angular speed of a permanent magnet synchronous motor based on a fractional-order PID sliding mode surface is presented. Remarkably, a mechanical system is again considered, similar to [110]. Along similar lines, in [149], a fractional-order PID sliding mode observer is developed. The take-away here is the actual industrial application of a fractional sliding mode control.

In [150], an invention is described that provides a formalization method to verify the stability of a robot where the control system comprises a FOPID controller. The author claims that the proposed method guarantees high reliability of the fractional-order PID control system and thus provides a solid foundation for the safety of human-robot interaction.

The industrialization of the FOPID controller clearly relies on a trust in this controller, namely, on its implementation and reliability. To increase that trust, at least three issues should be focused on in further research:

1) Provide simple tuning rules that are parameterized via required loop robustness (e.g., maximum value of sensitivity function) and available bandwidth. Moreover, those rules should be easily implementable into the controller function block and work on-line.
2) Trust in implementation, i.e., create reliable approximation of fractional elements that are well tested; here, the recent research in FOPID fragility seems to be promising.
3) Define clear conditions when fractional control is beneficial, compared to traditional PID, namely, its 2DOF version. There is still a lack of such a type of rigorous research.

It was recently verified in, e.g., [6], [38], that the technology suitable for actually physically implementing reliable FOPID controllers is readily available and that this technology stands against repeated laboratory tests. One important step for industrialization of FOPID control is clearly to move from laboratory experiments to actual field tests, i.e., to increase the TRL. As the analysis of the filed patents shows, there is potential to proceed with the integration of FO control to industrial applications. To accelerate the pace, the solution used in the field tests must be of the plug-and-play type, meaning that the FO control should be easy to install and replace the original PID control loop. We reflect on this aspect in the next section.

VIII. ANALYSIS, DESIGN TOOLS AND REALIZATION CONCERNS OF FO CONTROLLERS

An advancement in software tools that involves computer-aided analysis and design automation modules is a vital step for industrialization of FO control so that computer-aided control system design (CACSD) tools can provide reliable and fast FO system solutions without the user necessarily having extensive expertise in fractional calculus and FO system methodologies. The need for special expertise implies additional cost and time. Therefore, to accelerate the industrialization process of FO control—an essential point—relevant commercial initiatives notwithstanding, one should invest effort in the development of computer-aided design automation tools that are fully integrated with low-cost control cards via relevant integrated development environments (IDEs), leading to affordable realization of near-ideal fractional-order controllers and becoming a competitive solution for industrial use instead of low-cost PID solutions. Numerous analyses, simulations and implementation methods and software tools that are developed for research purposes of FO control systems in an open-access and nonprofit manner are summarized below.

A. A SUMMARY OF STABILITY ANALYSIS METHODOLOGIES FOR FRACTIONAL-ORDER SYSTEMS

Since unstable system solutions are mainly useless for control practice, a major concern in controller design is the stability of designed control systems; as we have seen, even patents such as [150] are specifically targeting this issue. Methods for checking system stability have become a fundamental requirement imposed on controller design tools. The theoretical background for stability analysis of

fractional-order systems is, by this point, sufficiently well-established in the literature. Today, computer-aided stability analysis can be performed consistently. Robust stability and stability ranges of system parameters are also important for optimal controller design tasks. Therefore, in this part, we reflect on the literature that specifically deals with fractional-order system stability. The review is also valuable for tasks related to optimal control design tool development.

Placement of system poles provides useful knowledge on system behavior and robust stability. Stability checking based on system pole placements were discussed in [151], [152] , [153], [154] and [155], and optimal stabilization based on minimum angle pole placement in the v-domain were shown in [156], [157].

FO controller stabilization based on stability boundary locus (SBL) analyses [158], [159] has significance, particularly for determining search ranges of metaheuristic tuning methods and thus, contributes to the development of optimal controller design tools.

The stability test based on a value set [160], [161] can be helpful, particularly when exploring whether the designed control system is meeting robust stability range specifications.

The linear matrix inequalities (LMI) method [162]–[164], of graphical stabilization methods for robust stabilization of interval plant functions [165], [166] and robust FOPID controller design by using the D-stability method [167] are other methods considered for checking the stability of a given fractional-order system.

B. BRIEF INTRODUCTION OF APPROXIMATE REALIZATION OF FRACTIONAL-ORDER SYSTEMS

Fractional-order elements such as the fractional-order derivative and integrator are nonlocal operators, and they have numerical dependence on the past values of the fractionally derived functions. Although this property brings the advantages of better characterization of real-world phenomena, the property causes high computational complexity when an ideal FO element implementation is sought for engineering use. To ensure correct computation of fractional operators, the memory capacity that is necessary for storing past values of parameters and functions increases. Therefore, ideal or near-ideal realization of fractional-order elements in a digital system can seriously consume computational resources in the long run. This effect is popularly known as the long-memory effect, and it is a fundamental issue for digital implementation of FO systems, which largely increases realization costs in the case of the near-ideal FO controller realization and waiting for a genuine mathematical solution. Nevertheless, instead of using ideal realization, approximate integer-order models of FO elements have frequently been used in digital systems and FO system simulations. As we have seen, this situation immediately raises a plausible question of what the use of an FO controller is, if it is implemented by approximate high-order IO functions; see Section V for the relevant discussion. Some fundamental digital approximate realization methods were explained in [40], [55], [168], [169]. However, the superiority of FO controllers to high-order IO controllers can be entirely harvestable in the case of near-ideal realization of FO elements instead of approximate IO realization. These complications in digital realizations lead to research interests that target analog realizations of FO elements. Analog integrated circuit (IC) realizations and FO element solutions from the field of engineered materials can result in practical and effective solutions to obtaining low-cost near-ideal FO elements. We consider several related technologies in the next subsection.

Today, the vast majority of FO control-related research studies are still based upon approximate realizations of FO models. Approximate equivalent transfer functions can provide acceptable approximation to the response of ideal FO elements in limited operating frequency ranges for digital implementation and simulation purposes of fractional-order control systems. Some useful details on fundamental approximate realization methods were elaborated in survey works [40], [168], [169]. A few fundamental and contemporary approximation methods based on use of s-domain integer-order transfer functions are listed in Table 3.

To the best of our knowledge, there is also a need for research studies that can suggest application-driven selection criteria for discrete-time and continuous-time realization methods of FOPID controllers. The experimental research knowledge that can demonstrate control performance concerns of discrete-time or continuous-time FOPID controllers and provide selection criteria based on application types will be helpful for implementing FOPID control in industrial applications.

C. PROSPECTS FOR DEVELOPMENTS IN LOW-COST ANALOG AND INTEGRATED CIRCUIT REALIZATIONS

Researchers in the field of FO systems foresee that development of a low-cost fractional-order control card with a support of IDE software will stimulate industrial control communities to consider the FOPID controller as an industrial control standard. Such an all-in-one commercialization attempt offers opportunities for leading control system producers and system-on-a-chip (SoC) enterprises in addition to accelerating the impact on the industrialization of FO controllers.

Recent research efforts make progress in the use of analog implementation of approximate FO elements (FOEs). A short survey of related studies on analog realization of FOEs was presented in [40]. Specifically, integrated circuit implementation will be the most likely solution for the low-cost implementation problem of FO elements because of its mass production assets. A brief survey of analog integrated circuit (IC) realization of an approximation method was provided in [184], [185]. To improve the versatility of IC designs of FO elements, the following properties are important to consider in design stages:

TABLE 2: Lists some statistical parameters that can be provided in comparative works involving meta-heuristic optimizations

Statistical Parameters	**Explanations and Recommendations**
Number of repeated tests	There is a need for statistically meaningful repeated test counts. We recommend doing at least 100 tests to present acceptable results. Test run counts should be revealed in publications.
Initial ranges of tuned parameters	For reliable comparison, initial range configuration of meta-heuristic methods should be kept the same and these ranges should be revealed in publications.
Average values of test results	Average values of controller parameters and performance metrics should be provided for each compared method and controller. These average values of parameters express expected value of performance indices.
Standard deviation of test results	Standard deviation of controller parameters and performance metrics should be provided for each of the compared methods and controllers. It is useful to evaluate consistency of results.
Maximum and minimum values of test results	Maximum and minimum values of controller parameters and performance metrics should be provided for each of the methods and controllers. It is useful to reveal the best-case as well as worst-case performances.
Computation time	For many real applications, computational complexity is a major concern and limitation. Therefore, computation time should be provided in publications.

1) The analog IC design should have electronically configurable fractional orders. This property is essential for development of reconfigurable control cards that are supported by IDE tools. In this case, an IDE tool can electronically reconfigure fractional orders of the control card. For an easy-to-use design-and-development environment, following the design of the FO controller by means of the IDE tool, the controller should be uploaded to a control card (See Figure 4 for an illustration).
2) Operating frequencies of industrial control applications generally lie in the lower range, and approximation performance of analog fractional-order IC designs should be effective in low-frequency regions, typically in the range of 0–1 kHz.
3) Proposed IC design should not need changes in peripheral resistors or capacitor components when the fractional order or DC gain of fractional-order derivative elements are changed.
4) High-input impedance and low-output impedance of IC is required to facilitate integration for real systems.

SoC realization of FO elements has a strategic importance for portability and cost efficiency. Some important progress can be summarized as follows.

Experimental demonstration of on-chip CMOS fractional-order capacitor emulators that were designed in Austria microsystems (AMS) 0.35 μm CMOS technology [186] encourages commercialization efforts of FO elements for use in industry. Electronically configurable, CMOS 0.35 μm technology, operational transconductance amplifier (OTA)-based analog fractional-order controller realization was presented in [187]. The approximation range of this design was given as 0.1 Hz and 1 kHz, and its application to DC motor control was shown in the study. In a recent work, a FOPID controller was implemented by using electronically controllable constant phase blocks formed by bilinear transfer segments [184]. This design was implemented using modified current differencing units (MCDUs) that were designed for CMOS 0.18 μm technology. Its operation in the 1 kHz–100 kHz frequency range was demonstrated by simulation results. This kind of analog IC design can be adopted for a mixed-mode IC design, where fractional-order operations are carried out by analog elements while the rest of the functionality such as interfacing and management of analog parts and integration of these parts to an IDE tool can be carried out by the digital part. At present, such mixed-mode IC design concepts can open doors on the way to realization of low-cost, fractional-order control cards with IDE tools, which is very important for productization, commercialization and industrialization of fractional-order controllers. In the analog realization domain, there are also approximate implementations of fractional elements by using discrete active and passive elements such as RC or RL ladder structures [188]–[193]. A brief survey of FO element realizations in term of multicomponent and single component solutions was presented in [194]. These works clearly demonstrate that a fractional-order circuit design using fractional-order passive elements can be a realistic solution for analog FOPID controllers.

Another research outcome for implementation of fractional-order behavior is in the progress of the material science and micromechanical structure domain. These groups of studies rely on exploitation of the inherent fractionality in the nature of existing and engineered materials. The main challenge is to develop a methodology that can yield a configurable fractional-order behavior for practical use in the control engineering domain. We next review a few relevant papers.

Direct realization of FOEs by discrete elements has been demonstrated for polymethyl methacrylate [195], [196] and ionic polymer metal composites [197]. There are also noteworthy efforts for distributed resistive-capacitive realizations of fractional-order elements by using thin-film technology. For discrete FOE realization, significant and promising solutions for using resistive-capacitive elements with distributed parameters (RC-EDPs) have already been presented

TABLE 3: List of some continuous approximation methods that can be used for approximate realization of fractional-order elements in the s-domain

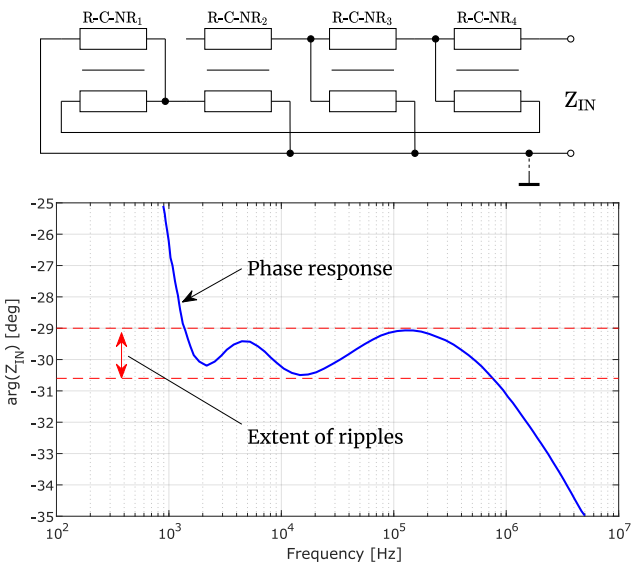
Approximation Methods	Explanations	Related works
Oustaloup's Method	This method is widely regarded as a fundamental approximation method in FO control. Oustaloup's method is based on using cascaded integer-order zero-pole filter series, where zero and pole frequencies are properly selected to obtain approximation to frequency response of FO elements in a given frequency range. As its approximation performance decreases at the boundaries of the frequency range, a modified version of Oustaloup's method was proposed by Xue et al. [40], [170].	[170]–[172]
Carlson's Method	This method benefits from Newton's iterative solution to approximate the fractional-order function [40], [168]. Some limitations of Carlson's method were resolved by Shrivastava et al. in [173] and Tepljakov et al. in [174]. Tepljakov et al. modified Carlson's method to obtain an approximation in a user-defined frequency range [174].	[173]–[175]
Continued Fraction Expansion (CFE) Method	This method is based on the use of a continued fraction expansion formula to express integer-order series expansion of s^α. The method does not allow for specifying the frequency range, and the resulting integer-order functions provide an approximation in the low-frequency region.	[171], [176], [177]
Matsuda's Approximation Method	This method is an extension of the CFE method by using logarithmically spaced set points of s_i, which allows manipulation of frequency points to be approximated.	[171], [178]
Charef's Method	This method is also based on cascaded filter bank realization by properly selecting zero and pole frequencies and is similar to Oustaloup's method [40] and modification for cases where low filter orders are needed [179].	[169], [171], [180]
Some contemporary methods	— An approximation method based on matching of the stability boundary locus (SBL) was proposed by Deniz et al. for control system implementations to better preserve the stability properties of the original fractional-order system and its integer-order approximation [181]. — Yuce et al. used time response of first-order transfer functions to approximate impulse response fractional-order elements [40], [182]. — Aware et al. presented a method that provides an approximation to phase response within a tolerance value by configuring pole and zero frequencies of a filter bank [40], [183]. This method is similar to Oustaloup's method. However, the method allows controllable phase response approximation.	[181]–[183]

in [198], [199] and [200]. By using the RC-EDPs, direct implementation of discrete fractional capacitors has been shown by providing layout design tools [192]. This paper reveals a remarkable realization of a fractional capacitor in a wide frequency range in $[1\ \text{kHz}, 1\ \text{MHz}]$ with an allowed ripple in the range of degree in phase response by configuring the layouts of R-C-NR networks (see Figure 3a) and proves that obtaining a low-cost, near-ideal FOE is possible. Another realization of fractional-order elements based on engineered materials was suggested by dipping a capacitive-type probe into a polarizable medium. The authors explained the natural origin of fractional behavior as "*the FOE originating due to the diffusion of ions through the micropores of the coating surface. The geometry of the microporus film is fractal in nature*". Figure 3b shows an SEM image of the micropores of the coating surface that leads to fractional-order behavior [201]. Very low-cost, near-ideal realization opportunities of FOEs may come from research efforts in the engineered structures and materials field, and the corresponding outcomes may contribute to control engineering practice by introducing low-cost FOE components with the desired fractal characteristics.

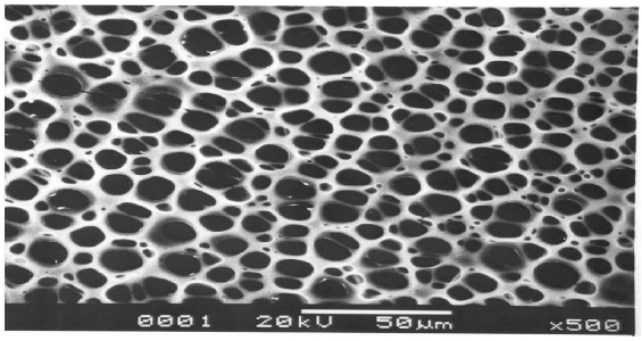
Today, we are at the dawn of the transformative technologies appearing for the future of energy systems: technologies that make possible sustainable and ecological energy utilization and electrification of transportation. An important application of fractional-order modeling and control comes in modeling and management of energy storage systems, e.g., batteries, supercapacitors and fuel cell technologies [202]. These components are key technologies at the core of future energy systems and growing electric vehicle sectors and electrified transportation. Fractional calculus and fractional-order modeling have useful implications in the development of these technologies by providing analysis and modeling [202]–[204], parameter estimation [204], [205], and state monitoring [206].

D. ANALYSIS AND DESIGN TOOLS DEVELOPED FOR FRACTIONAL-ORDER SYSTEMS:

To develop IDE software, there is a need for proven software toolbox functions and model libraries for carrying out the tasks of design, simulation and verification of fractional-order systems. IDE software will be very versatile when it has the capability of online modeling from system data captured by the low-cost control cards, including optimization tools for FO controller design and control system simulation environments for numerical verification of the design immediately before loading the designed optimal FO controllers to the control card (See Figure 4). The IDE software for fractional-order control cards can draw from several useful MATLAB toolboxes for analysis, modeling, simulation and design of fractional-order systems that have been developed and delivered independently in different periods by different researchers for nonprofit research use. These toolboxes can



(a) RC-EDPs by using R-C-NR networks (top) and its phase response (bottom) [192]



(b) SEM image of the micropores of the coating surface [201]

FIGURE 3: Fractional circuit example. The phase response of the circuit is flat in a broad frequency region spanning from 1 kHz to 1 MHz, with the extent of phase ripples of approximately $\pm 1^\circ$.

be utilized as proofs of the required modules for fractional-order control IDE software. The development process of a commercial IDE software can be facilitated significantly by considering those toolboxes. The widely used toolboxes and some contemporary tools are briefly introduced in Table 4.

IX. CONCLUSIONS

In this paper, a comprehensive review of literature related to the industrial use and integration of FOPID control was presented. Key issues related to the process of industrialization of FOPID controllers were identified and elaborated upon. Based on the evidence reviewed in the present paper, the following conclusions can be drawn.

With two additional "tuning knobs", FOPID controllers offer clear advantages over conventional IOPID controllers as seen from both simulation and experimental studies with real-life objects. The comparison results are solidly based on global optimization-based tuning for both types of controllers. Heuristic FOPID tuning methods can be considered attractive due to the relative simplicity of implementing the corresponding algorithms in very complex tuning problems but do suffer from issues related to the assessment of results. For fair evaluation of results of heuristic tuning methods, statistical assessment norms are recommended to be considered in research publications. Overall, heuristic tuning methods can serve best for the implementation of automatic tuning algorithms, which means that industrial partners need not concern themselves with the underlying complexity. Moreover, heuristic tuning methods allow tuning of a high-order integer-order controller, the starting point for which can be the FOPID controller, making the heuristic tuning methods very attractive in these applications.

The most common design method for fractional-order controllers is based on frequency-domain analysis. The properties of Bode's ideal transfer function and the isodamping property are essential characteristics that define "*fractal*" robustness and showcase the advantage of fractional-order controls. From CRONE controllers to FOPID controllers, the basic ideas and advantages that fractional components offer are essentially the same. However, the advantage of FOPID controllers is that they will still be more familiar to industrial practitioners since they are just an extension to conventional PID controllers. With the latter having an almost 80% market share in control loops across the industry according to some statistics, this makes it particularly advantageous to simply update PID controllers to FOPID controllers having these favorable dynamical characteristics, especially since methods have been developed to introduce the dynamics of a FOPID control loop into an existing PID control loop essentially without modifying the latter and hence without any process interruptions.

One significant advantage of FOPID controllers when applied to industrial problems is the potential of reducing the control effort, which, in turn, can result in a reduction in wasted energy of various control processes. This very important point has been continuously brought up in the literature. If the additional tuning flexibility of FOPID controllers allows modest improvement in the control law to reduce energy waste locally, globally—when all small improvements are combined—this should lead to a significant improvement.

An ideal "*basic building block*" implementation of FO

TABLE 4: Widely used MATLAB toolboxes targeting fractional-order modeling and control and some contemporary tools

Name of Toolbox	Brief Intoduction	References and Useful Links Related Toolboxes
CRONE toolbox	CRONE toolbox is one of the first toolboxes to emerge following several decades of research and development performed by the CRONE team specifically targeting fractional-order modeling and control. The CRONE toolbox is offered in two versions: the classical version is supported by graphical user interfaces, and the object-oriented version is best suited for programmatic development. Both versions offer tools for general modeling, system identification, and controller design. While the toolbox is built upon proven research, it is not open-source and is governed by a fairly restrictive license, which serves as an obstacle for its efficient adoption in both academia and industry.	[207] Website: http://cronetoolbox.ims-bordeaux.fr/
Ninteger Fractional control toolbox for MATLAB	Ninteger is a toolbox for MATLAB intended to help to simulate fractional-order controllers and assess their performance [208]. It contains several digital and continuous approximation methods. Simulink models for FOPID implementation and fractional derivative elements are provided in the toolbox.	[208], [209] Website: https://fenix.tecnico.ulisboa.pt/homepage/ist14074/matlab-toolbox-ninteger https://www.mathworks.com/matlabcentral/fileexchange/8312-ninteger
FOTF Toolbox	FOTF, designed to be a standard toolbox for fractional calculus and fractional-order control, contains the following: (i) algorithms for computing high precision fractional derivatives, (ii) FOTF and FOSS classes to fully support the modeling, analysis and design of multivariable fractional-order systems, and (iii) Simulink blockset and fotflib to allow modeling and simulation of fractional-order systems with any complexity. FOTF is a very comprehensive toolbox, and it is very useful in performing Simulink simulations of fractional-order systems. This toolbox is announced to be a supportive material of a book (Fractional-order Control Systems - Fundamentals and Numerical Implementations) [210]	[210], Website: https://www.mathworks.com/matlabcentral/fileexchange/60874-fotf-toolbox
FOMCON Fractional-Order Modeling and Control	FOMCON, developed to facilitate design and modeling of fractional-order control systems, provides an integrated modeling and simulation environment utilizing, to some extent, features found in several other toolboxes such as the CRONE toolbox from MATLAB [207], Ninteger toolbox [208], FOTF toolbox [210]. The FOMCON toolbox is composed of a main module for fractional-order system analysis, an identification module for time and frequency-domain identification, and a control module for FOPID tuning and optimization problems. The FOMCON toolbox also has a digital and analog FO element implementation module. The module supports both analog circuit design with RC(L) networks and infinite impulse response (IIR)-based digital implementation, with a graphical user interface for beginners, while the full API is available to experienced MATLAB programmers to create their own applications. A Python implementation of the core library is currently being worked on [211].	[212] Website: https://www.mathworks.com/matlabcentral/fileexchange/66323-fomcon-toolbox-for-matlab https://fomcon.net/
FLOreS Fractional-order LOop-Shaping Toolbox	FLOreS was developed to perform loop-shaping design for FOPID control systems. FLOreS has a user-friendly graphical interface to facilitate frequency-domain controller design tasks and accepts integer-order or fractional-order SISO plants, performs design and tuning of both integer-order and fractional-order controllers.	[213] Website: https://www.tudelft.nl/en/3me/departments/precision-and-microsystems-engineering-pme/research/mechatronic-system-design-msd/msd-research/motion-control/fractional-order-loopshaping-toolbox/

operators should be sought to compensate for the high-order approximation issue. This implementation must be cost-effective to facilitate industrial adoption of FO controllers. Analog or mixed mode analog-digital integrated circuit solutions can be useful to realize near-ideal fractional-order elements or transfer functions at a low cost for development of inexpensive FO controller cards. For industrial and commercial use, all-in-one IDE tools, which facilitate design, simulation, integrity, verification, realization and monitoring tasks for users, are very substantial because the cost of developing an expertise in fractional calculus and fractional-order system design may be a discouraging factor while there is an available and working infrastructure of conventional IO control.

Even if an ideal implementation of a fractional operator is not available—a point that is still being debated in the fractional calculus scientific community—approximation-based implementations have been used for many years, and research efforts spanning many decades show that FOPID technology is reliable and ready for use in industrial applications. Several approximation methods have been reviewed in this paper. We can conclude that these methods draw mostly from the same source—a geometric distribution of zeros

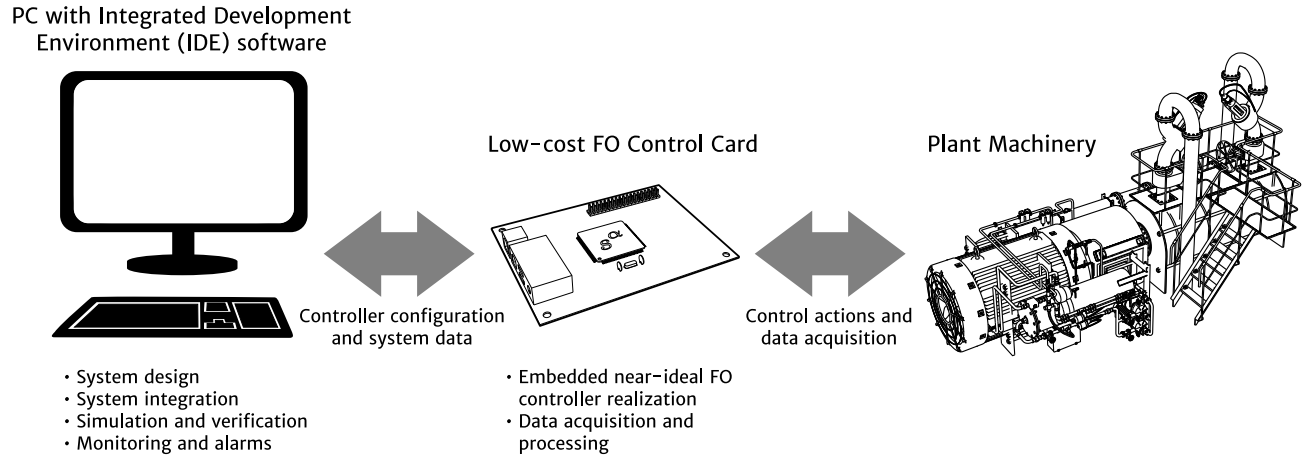


FIGURE 4: Schematic diagram of fully integrated fractional-order control systems that include analysis, design, implementation and verification environments.

and poles—and differ only in the way that this distribution is computed. The Oustaloup approximation method still dominates this space providing simple, clear relationships for quickly and efficiently obtaining an approximation of an FO operator, system, or controller in the given frequency band. FPGA-based digital realization of the FO element was shown for DC motor control [214], and the results encourage design of FO control cards for industrial use.

The area of fractional control is well supported by scientific software in the form of so-called toolboxes, the more popular of which were reflected upon in this paper. Observing the download statistics for these tools, one can conclude that the interest in using fractional modeling and control software tools is ever-increasing. Developers of industrial software solutions involving plug-and-play fractional control blocks should consider these tools since they are built upon years of relevant experience in identification and control design for FO systems.

Concerning industrialization, several patents were reviewed. The patents that deal with FO control technology directly appear to be less successful. Perhaps the time frame for filing these patents was not ideal. However, the patents that deal with inventions that simply involve FO controls in some way appear rather frequently and appear to be successful. Hence, we envision that many more patents, both local and international, that contain FO technology as part of the invention, will be filed in the coming years.

Industrial communities have a right to request comprehensible explanations for the role of the fractional-order derivative and the integral operant in control actions before investing in them. From the control system point of view, one can emphasize effects of the nonlocality property of the fractional order differentiation on control actions of closed-loop control systems. More specifically, fractionalization of derivative elements by using a FOPID controller results in implementation of a nonlocal time derivatives of control error in control actions. This modification allows benefiting from the previous changes in the control error through the past states of systems when generating control actions for the current states. In contrast, an integer-order derivative is a local operator that can consider only current changes in the control error to generate control actions. However, an integer-order integral is a nonlocal operator that sums up all previous values of control errors with an equal weighting, whereas a fractional-order integral allows the weighted sum of all previous control error values while generating the current control actions. By adjusting the fractional integral order, a FOPID controller can increase weighting of very recent error values relative to the past values in error accumulation of an integral element. Such tunable flexibilities in treatment of the FO derivative and integral elements are the major factor leading to improvement of FOPID controller laws for a wider range of controlled system dynamics than those of a PID controller.

Several proposals were made in this paper based on the findings related to the key issues. Overall, we can conclude that FO technology is mature and is ready for large-scale industrial integration.

The global transition from PID to FOPID controllers is seen as natural due to the similarities in these controller types, with the latter simply offering more tuning freedom that allows achievement of more design specifications. Future research efforts in the area of FOPID controller industrialization should be directed at developing the tools and guidelines to facilitate this transition and at providing more successful use-case examples of industrial use of FOPID controller technology.

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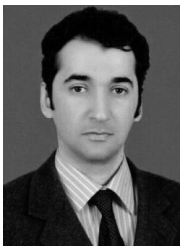

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